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Control of tunneling in heterostructures

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Abstract
A tunneling current between two rectangular potential wells can be effectively controlled by applying an external ac field. A variation of the ac frequency by 10% may lead to the suppression of the tunneling current by two orders of magnitude, which is a result of quantum interference under the action of the ac field. This effect of destruction of tunneling can be used as a sensitive control of tunneling current across nanosize heterostructures.

1. Introduction

The idea of controling quantum tunneling through a potential barrier by an external nonstationary field has a long history. Initially, the problem was addressed in [1, 2] for atoms ionization in an ac field. The problem was further developed in [3–6] by the method of complex classical trajectories. Achievements in the study of tunneling through nonstationary barriers are presented in [7–19]. In [20–22] the approach was developed to go beyond the method of classical trajectories and to obtain the space–time dependence of the wavefunction in the semiclassical regime. Some of experimental investigations of tunneling through nonstationary barriers are presented in [23–25] where Josephson junctions were studied.

In [26] the smooth double-well potential in an external ac field was considered. It was shown that due to interference effects under certain conditions the tunneling rate was substantially reduced. This effect of destruction of tunneling was studied further in [27] for a specific case of a two-level system.

The goal of this paper is to consider the above type of control of tunneling for the case of two rectangular wells separated by a thin potential barrier. This particular choice of the potential corresponds to tunneling in artificial heterostructures used in nanophysics. We solve numerically the Schrödinger equation. It is shown that one can effectively manipulate with the tunneling current across the sandwich by a weak variation of the amplitude and frequency of the applied ac field. This constitutes a possible method of quantum control in nanostructures.

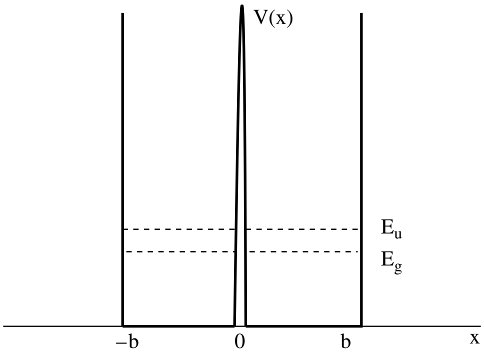

Figure 1. The static part of the potential relates to a double-well.

2. Formulation of the problem

We consider the one-dimensional Schrödinger equation

$$\mathrm{i}\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + [V(x) + ax\sin\omega t]\,\psi, \tag{1}$$

where the potential $V(x)$ is defined as

$$V(x) = \begin{cases} \infty; & \mathrm{b} < |x| \\ \lambda\delta(x); & |x| < b. \end{cases} \tag{2}$$

The constant λ is positive. The static part of the total potential, $V(x)$, is shown in figure 1. In the absence of the nonstationary component ($a = 0$), discrete energy levels $E = k^2/2$ are determined by the equation

$$\frac{\tan kb}{kb} = -\frac{1}{\lambda b}. \tag{3}$$

In the limit of an almost non-transparent δ-barrier

$$1 \ll \lambda b \tag{4}$$

the ground state is characterized by the eigenfunction and the energy

$$\psi_{\mathrm{g}}(x) = \frac{1}{\sqrt{b}}\left|\sin\frac{\pi x}{b}\right|, \qquad E_{\mathrm{g}} = \frac{\pi^2}{2b^2}\left(1 - \frac{2}{\lambda b}\right). \tag{5}$$

For the first excited state

$$\psi_{\mathrm{u}}(x) = \frac{1}{\sqrt{b}}\sin\frac{\pi x}{b}, \qquad E_{\mathrm{u}} = \frac{\pi^2}{2b^2}. \tag{6}$$

3. Analytical approach

In the limit of the hardly transparent barrier (4) one can use an approximation of two levels (5) and (6)

$$\psi(x,t) = a_{\mathrm{g}}(t)\psi_{\mathrm{g}}(x) + a_{\mathrm{u}}(t)\psi_{\mathrm{u}}(x), \tag{7}$$

where the functions a_{g} and a_{u} obey the equations

$$\begin{aligned} \mathrm{i}\frac{\partial a_{\mathrm{g}}}{\partial t} &= E_{\mathrm{g}}a_{\mathrm{g}} + K a_{\mathrm{u}} \sin \omega t \\ \mathrm{i}\frac{\partial a_{\mathrm{u}}}{\partial t} &= E_{\mathrm{u}}a_{\mathrm{u}} + K a_{\mathrm{g}} \sin \omega t. \end{aligned} \tag{8}$$

Here the parameter

$$K = \int_{-b}^{b} \mathrm{d}x\, \psi_{\mathrm{u}}(x) a x \psi_{\mathrm{g}}(x) \tag{9}$$

is introduced. In the limit (4) the parameter (9) is given by

$$K = \frac{ab}{2}. \tag{10}$$

For the function

$$g = \ln \frac{a_{\mathrm{g}}}{a_{\mathrm{u}}} \tag{11}$$

it follows from equations (8) that

$$\frac{\partial g}{\partial t} = \mathrm{i}\Omega + 2\mathrm{i}K \sin \omega t \sinh g, \tag{12}$$

where $\Omega = E_{\mathrm{u}} - E_{\mathrm{g}}$. In the limit (4) the parameter Ω is

$$\Omega = \frac{\pi^2}{\lambda b^3}. \tag{13}$$

We are interested in the case of an almost equal population of the two levels $a_{\mathrm{u}} \simeq a_{\mathrm{g}}$. In this case, according to equations (5)–(7), the particle is localized in the right-hand side well in figure 1. This condition is equivalent to $|g| \ll 1$ and equation (12) turns into a linear one

$$\frac{\partial g}{\partial t} = \mathrm{i}\Omega + 2\mathrm{i}K g \sin \omega t, \tag{14}$$

which has the solution

$$\frac{g(t)}{\mathrm{i}\Omega} = \exp\left(-\frac{2\mathrm{i}K}{\omega} \cos \omega t\right) \int_0^t \mathrm{d}t_1 \exp\left(\frac{2\mathrm{i}K}{\omega} \cos \omega t_1\right). \tag{15}$$

As follows from the solution (15), initially, at the moment $t = 0$, the particle was localized in the right-hand side well since $g(0) = 0$. The value of $|g(t)|$ can remain small when the integral over the period in equation (15)

$$\int_0^{2\pi} \frac{\mathrm{d}z}{2\pi} \exp\left(\frac{2\mathrm{i}K}{\omega} \cos z\right) = J_0\left(\frac{2K}{\omega}\right) \tag{16}$$

equals zero. In this case, the order of magnitude of $|g(t)|$ is no more than Ω/ω.

Now one can formulate conditions so that the particle is mainly localized in one well in figure 1 at all times

$$J_0\left(\frac{2K}{\omega}\right) = 0, \qquad \frac{\Omega}{\omega} \ll 1. \tag{17}$$

The first condition (17) is plotted in figure 2.

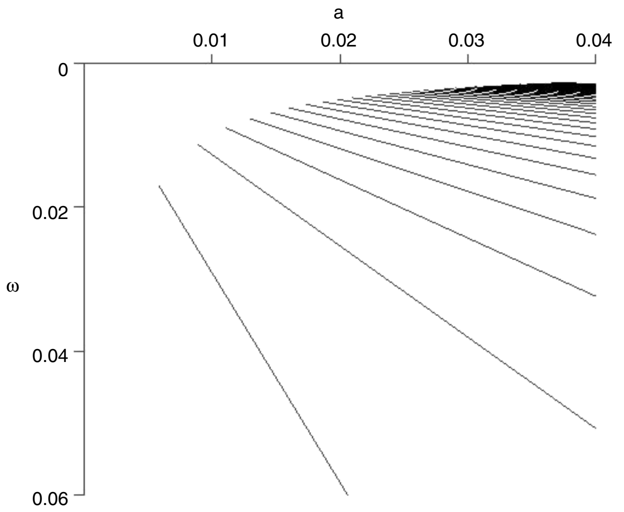

Figure 2. The straight lines, equation (17), correspond to destruction of tunneling within the approach of two levels.

4. Numerical calculations

We solve directly the Schrödinger equation (1) by numerical methods. We take the boundary condition

$$\psi(\pm b, t) = 0. \tag{18}$$

The initial condition has the form

$$\psi(x, 0) = \begin{cases} \sin(\pi x/b), & -b < x < 0 \\ 0, & x < -b, 0 < x. \end{cases} \tag{19}$$

This corresponds to tunneling from the left well to the right one. We consider the state to be decayed when the probability in the initial well is dropped by a factor of e.

In equation (2) the values $b = 7$ and $\lambda = 0.8$ are chosen. The parameters a and ω were varied. We used the implicit six-point scheme which conserves the normalization of the wave function. The δ-function is accounted for by the proper jump of $\partial\psi/\partial x$ at $x = 0$. The set of linear equations obtained was reduced to the triple-diagonal form. The run was stopped either due to the decay of the state reaching the calculation time of $t = 10^6$ or if the decay did not occur. The last possibility happened at some values of the parameters. The number of spatial points was 21 and the time step was 0.01. In this scheme the accuracy of the calculation of the decay time was less than 0.1%.

The results of the numerical calculations are presented in figure 3. The density of dots in figure 3 is proportional to the logarithm of the decay time T which is a time of e times reduction in probability to find the particle in the initial well. The tunneling probability is proportional to $1/T$. The step for parameters a and ω was 10^{-4}. It can be seen in figure 3 that there are certain curves in the plane of $\{a, \omega\}$ which correspond to 'frozen' tunneling. In figure 4 the logarithm of the decay time T is plotted as a function of ac frequency ω at the fixed ac amplitude $a = 0.01$.

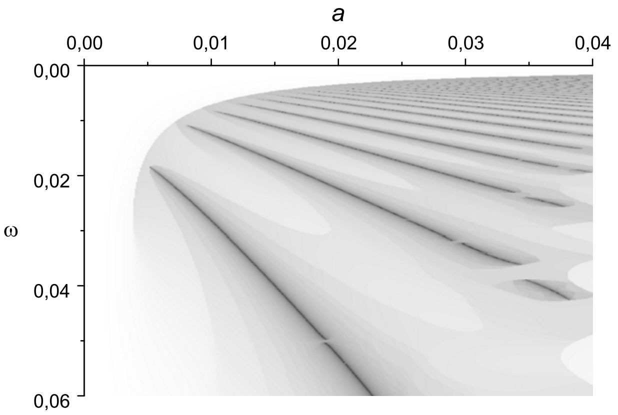

Figure 3. The results of numerical calculation on the basis of equation (1). The density of dots is proportional to the logarithm of the decay time T.

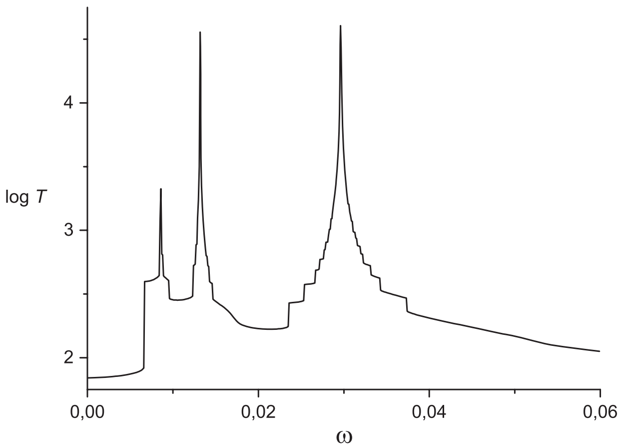

Figure 4. The dependence of the decay time T versus ω at the fixed ac amplitude $a = 0.01$. The peaks correspond to 'frozen' tunneling.

5. Discussions

In this paper we consider the influence of ac field on quantum tunneling. There are effects of chaotic motion in a shaken box [28] which are outside our attention at the moment. One should note also the phenomenon of the Kapitza pendulum [29] when a high frequency ac field can substantially modify an initial static potential (see also [30]).

Comparing figure 2 (two-level analytical approach) and figure 3 (a general numerical solution) one can conclude that both results are close to each other with the parameters chosen. This indicates that transitions through other levels are not significant. Indeed, the parameter λb is 5.6 which corresponds to the condition (4) of applicability of the two-level approach.

The condition (17) of 'frozen tunneling' does not depend on the strength of the potential barrier λ. Generally speaking, that is not true. This holds only in the limit (4) of the strong potential barrier we used.

One can apply the results obtained to tunneling across artificial heterostructures. Suppose the width b in figure 1 relates to 200 Å. Then the unit of coordinate x_0 should satisfy the relation $7x_0 = 200$ Å which results in $x_0 \simeq 2.8 \times 10^{-7}$ cm. The unit of time is $t_0 = mx_0^2/\hbar \simeq 0.78 \times 10^{-13}$ s. Let us chose in figure 3 $a = 0.01$ and $\omega = 0.03$. With these values the frequency is $\nu = \omega/2\pi t_0 \simeq 60$ GHz. The ac amplitude is $\mathcal{E} = \hbar a/x_0 t_0 \simeq 2.8 \times 10^2$ eV cm^{-1} which relates to the electromagnetic energy flux of 2.1×10^2 W cm^{-2}. Under those conditions the separation between energy levels is of the order of 0.3 meV. To observe the destruction of tunneling one should chose the temperature to be less than 4.2 K (0.36 meV).

We model the heterostructure as a rectangular potential. Obviously, this is not an exact approximation since in reality physical potentials are extended in space. Nevertheless, the final conclusion on 'frozen' tunneling is based on the fact that only two levels are mainly involved in the game. In this situation details of the potential shape are not crucial.

6. Conclusion

As one can conclude from figure 4, the tunneling probability, which is proportional to $1/T$ and determines a current across the barrier, is very sensitive to the ac amplitude and frequency. The frequency variation of the order of 10% results in suppression of the tuneling current by two orders of magnitude. This effect can be effectively used for quantum control of a current through heterostructures.

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